On-off intermittency in earthquake occurrence

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The clustered occurrence of earthquakes is viewed as an intermittent phenomenon, interpreting the clusters of events as chaotic bursts combined to the Poissonian occurrence of background seismicity. In particular, we suggest that it can be interpreted as an example of on-off intermittency. This kind of intermittency is parameter driven and exhibits certain universal statistical properties. The study of a Californian catalogue allows to interpret earthquake occurrence as an on-off intermittent phenomenon. Our results suggest the existence of a branching mechanism in earthquake occurrence well explained by epidemic type models.

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I. INTRODUCTION

Seismicity is a complex process featuring nontrivial space-time correlation in which several forms of scale invariance have been identified. One of the most intriguing aspects is the power law behavior of interevent time distribution [1–4]. As a consequence, earthquakes cannot be viewed as a Poissonian process which is characterized by a constant rate of occurrence and an exponential distribution of intertimes.

The clustering in earthquake occurrence has been worldwide observed and the well known main-aftershocks sequences can be viewed as bursts of activity. In his pioneering paper, Omori [5] investigated the problem of earthquake occurrence within a single cluster of events and proposed that the non-Poissonian behavior of seismic catalogues could be well fitted with the modified Omori law [6], stating that the number of aftershocks $n(t)$ decays in time as

$$n(t) = \frac{k}{(t+c)^p}, \qquad (1)$$

where p is generally very close to 1, ranging from 0.7 to 1.7, c is an initial time which avoids the divergence at $t=0$ and $k=n(0)c^p$ is an experimental constant.

More recently the non-Poissonian behavior of earthquake occurrence has been interpreted in terms of fractal geometry observing that a Poissonian process should exhibit a fractal dimension $\simeq 1$, whereas for a clustered one the fractal dimension is <1 [2,7–9].

The scaling properties of the earthquake temporal occurrence, well reproduced by the epidemic type aftershock sequence (ETAS) model [10,11], reveal a universal behavior [12–15], leading to consider earthquakes on the same foot level, independently by the tectonic features or the usual classification into main shocks and aftershocks.

Here we want to present an interpretation of seismic clustering in terms of on-off intermittency. The term "on-off intermittency" has been coined for some nonlinear dynamical systems exhibiting intermittency when the control parameter assumes a certain range of values. The intermittent behavior is characterized by no sharp transition from an equilibrium quiescent state (laminar phase) into an active state (burst). The name "on-off intermittency" derives from the characteristic two state nature of intermittent signals behavior: the on state and the off state, respectively, characterizing the burst and the laminar phase [16].

Recently, on-off intermittency has been widely studied experimentally and theoretically in many systems, e.g., a set of coupled ordinary differential equations and a driven piecewise linear map [16,17]; noise-driven electroconvection in sandwich cells of nematic liquid crystals [18]; synchronization-desynchronization of coupled identical chaotic oscillators [19]; coupled map lattice [20].

More recently [21] it has been obtained as a consequence of the mean-field approximation of self-organized critical (SOC) model with memory [22] for earthquake occurrence. According to this model we shall show that the clustered occurrence of earthquakes is a bursting phenomenon which can be interpreted in terms of on-off intermittency. In this approach, each "cluster" can be viewed as a burst (on state) whereas the waiting time between two successive "clusters" represents a laminar phase (off state). Notice that here a cluster of events is not intended, as usually done in traditional seismology, as a sequence main-aftershocks, but more generally as a "group" of correlated earthquakes.

II. ON-OFF INTERMITTENT EARTHQUAKE OCCURRENCE

Intermittency has been observed in a large variety of nonlinear dynamical systems. It occurs whenever the system appears to switch back and forth between two qualitatively different behaviors, even though all the control parameters remain constant and no significant external noise is present. The switching appears to occur, *randomly*, even though the system is described by deterministic equations. The classical intermittent bahvior can be found in deterministic systems characterized by fixed points that become unstable after some control parameter values are changed. The general scheme is that above given critical parameter values there is a transition to chaotic behavior.

Conversely, on-off intermittency is parameter driven and it is characterized by a power law distribution of the signal

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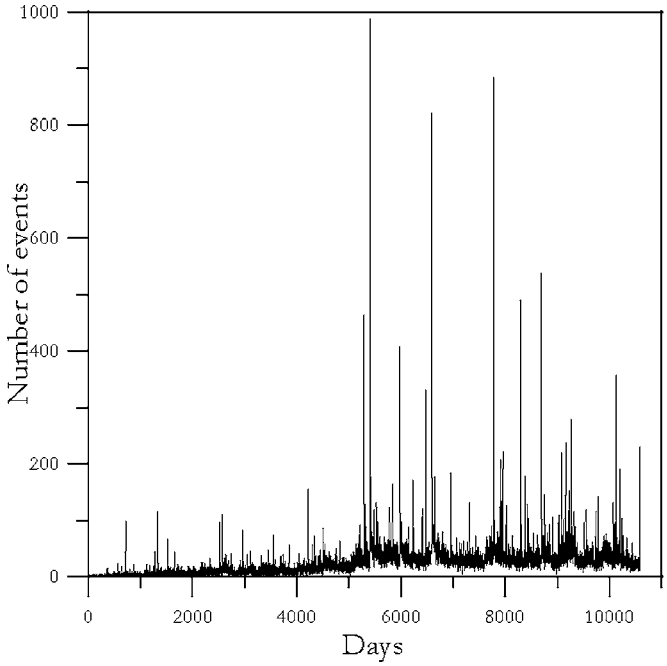

FIG. 1. Example of the signal $n(t)$ versus time for $\Delta t=1$ day for the Californian catalog.

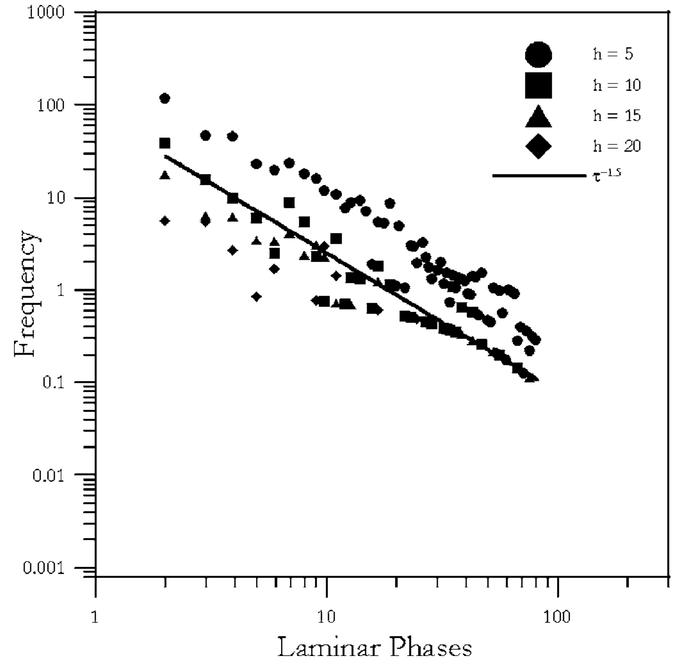

FIG. 3. Distributions of laminar phases duration for bin width $\Delta t=100$ min at different thresholds.

amplitude and of the laminar phases duration, whose exponent is equal to 1.5. This value is universal and independent of the choice of a threshold value defining the laminar phase (namely the signal is considered to be in a off state when it assumes values lower than the threshold) [16,17].

We analyze the Southern California Catalogue [23] in the period 1975–2003 composed by about 370 000 events with magnitude greater than 2.0 (different choices of this threshold value do not change significantly our results). The analysis is performed regardless of any tectonic feature and any classification in main shocks or aftershocks in order to unveil general statistical properties of earthquake occurrence.

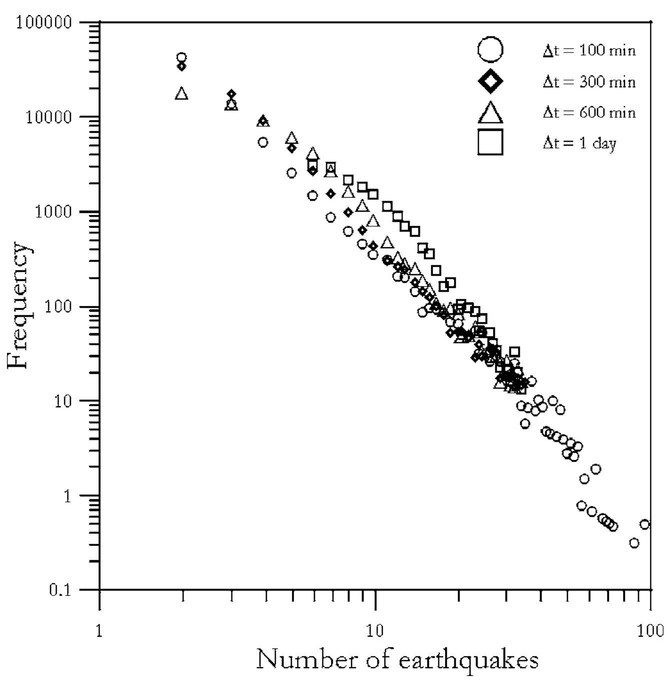

FIG. 2. Distributions of the earthquakes occurrence rate for the Californian catalog.

Let us consider the number of earthquakes n per time unit Δt versus the time t, we observe a signal $n_{\Delta t}(t)$ (Fig. 1) composed by bursts of seismicity followed and preceded by periods of quiescence. This is a widely observed feature in seismicity and it is mainly due to the temporal clustering of earthquakes [11,24,25]. We shall show that the distribution of the length of these quiescence periods between clusters of events follows a power law distribution very similar to the on-off intermittency one. In order to investigate the eventual dependence of the laminar phase distribution on the binning, we decided to analyze its behavior for different bin width Δt in the range 100 min $\leqslant \Delta t \leqslant 1$ day. More precisely we selected four values of the bin width: $\Delta t=100$ min, $\Delta t=300$ min, $\Delta t=600$ min, and $\Delta t=1$ day.

The distribution of series amplitude (in our case the analogous of the amplitude, for on-off intermittency, is the number of earthquakes per time unit) exhibits, as expected, power law behavior (Fig. 2) with an exponent α varying in the range 2.6–3.1 (Table I). A $t-$test reveals that these values can be considered equal within a confidence level of 5%.

TABLE I. Exponent of the amplitude distributions.

Δt	Exponent	Standard deviation
100 min	2.7	0.4
300 min	3.1	0.2
600 min	3.0	0.3
1 day	2.6	0.7

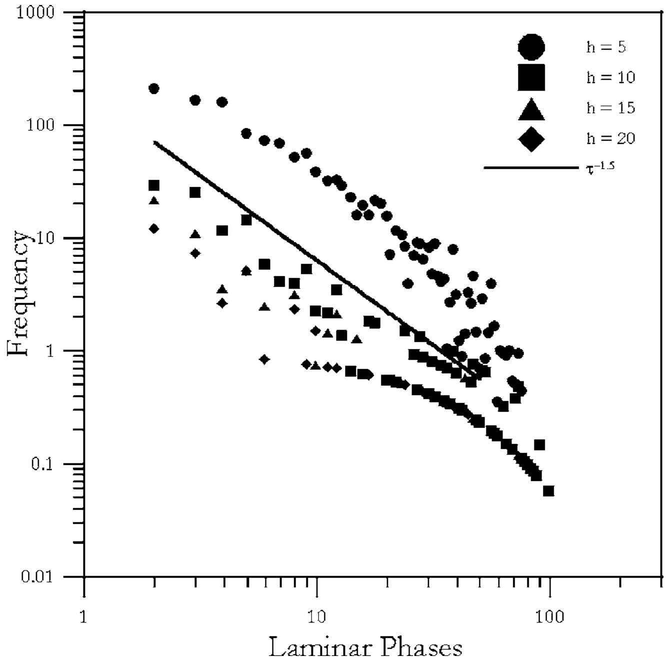

FIG. 4. Distributions of laminar phases duration for bin width $\Delta t=300$ min at different thresholds.

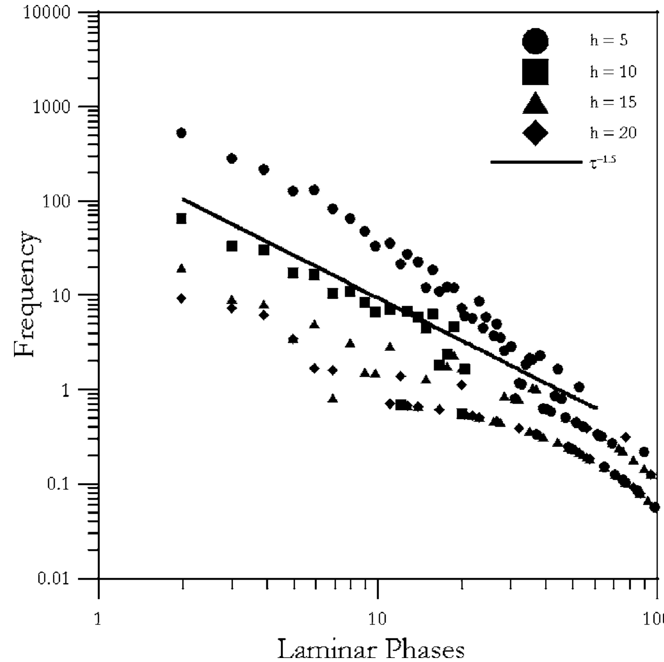

FIG. 5. Distributions of laminar phases duration for bin width $\Delta t=600$ min at different thresholds.

Therefore we decided to adopt the average ($\bar{\alpha}=2.8$) as the "true" value, which is in good agreement with some previous estimations [28,29], whereas it differs from the value found by Corral [13]. However, his distributions are evaluated for a smaller region and different time intervals. The range in which we observe a power law behavior decreases as the bin width increases: a lower cutoff in the the observability of the smaller laminar phases appears, because small clusters are absorbed in bigger ones and they disappear from the distribution.

Power law distribution of the earthquakes rate of occurrence is a very clear and well known signature for their temporal clustering [3,26,27]. Moreover it suggests the existence of a branching mechanism in earthquake occurrence. Indeed in the ETAS model, which can be considered as a benchmark in the explanation of many empirical observation, any earthquake is a potential progenitor of a certain number of children earthquakes. The power law behavior of the earthquake rate of occurrence is a direct consequence of the power law distribution of the conditional average number of children of first generation in this branching mechanism [28].

Now let us analyze the distribution of laminar phases length as a function of the threshold value h. The signal is considered to be in a off state when it assumes values lower than the threshold. Here h has been selected in the range 5–20 number of events per unit time. This choice derives from the observation that, if $h<5$, the laminar phases become too short and the phenomenon cannot be identified as intermittent anymore. Whereas, if $h>20$ the statistics becomes too poor and the laminar phases become too long, implying the loss of scaling invariance.

In the Fig. 3 we can observe the laminar phases distribution for a bin width $\Delta t=100$ min. Table II reports the values of the slopes which are all compatible with on-off intermittency within a confidence level of 5%.

When the bin width assumes the value $\Delta t=300$ min (Fig. 4) the exponents, reported in the Table III, are again compatible with on-off intermittency within a confidence level of 5%.

For bin width $\Delta t=600$ min (Fig. 5) the slopes are compatible with on-off intermittency, within a confidence level of 5%, except in the case of $h=5$ (see Table IV). We shall discuss the implications of the result for this h at the end of the section, enlightening the connection with chaotic behavior.

In the last examined case, characterized by bin width $\Delta t=1$ day, we observe (Fig. 6) the same feature of bin width $\Delta t=600$ min (Table V).

TABLE II. Exponent of the laminar phases distributions for $\Delta t=100$ min.

Thresholds	Exponent	Standard deviation
5	1.6	0.6
10	1.4	0.5
15	1.4	0.5
20	0.9	0.8

TABLE III. Exponent of the laminar phases distributions for $\Delta t=300$ min.

Thresholds	Exponent	Standard deviation
5	1.5	0.4
10	1.4	0.6
15	1.4	0.5
20	1.4	0.5

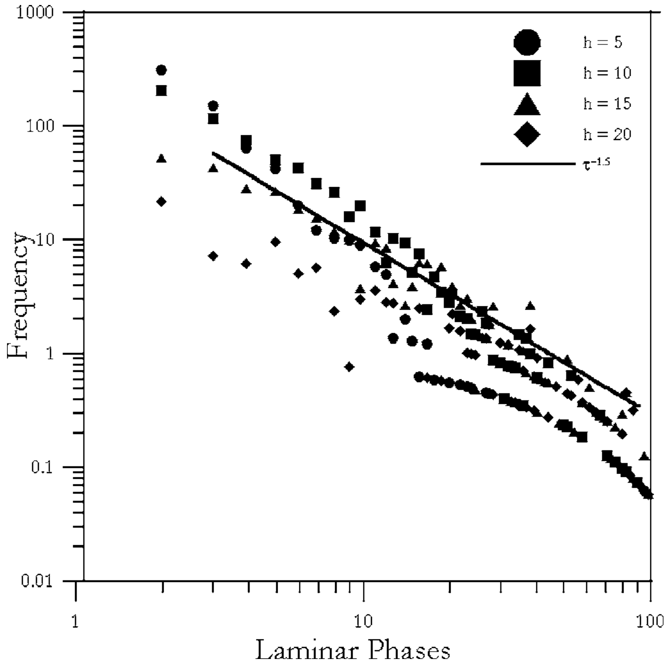

FIG. 6. Distributions of laminar phases duration for bin width $\Delta t = 1$day at different thresholds.

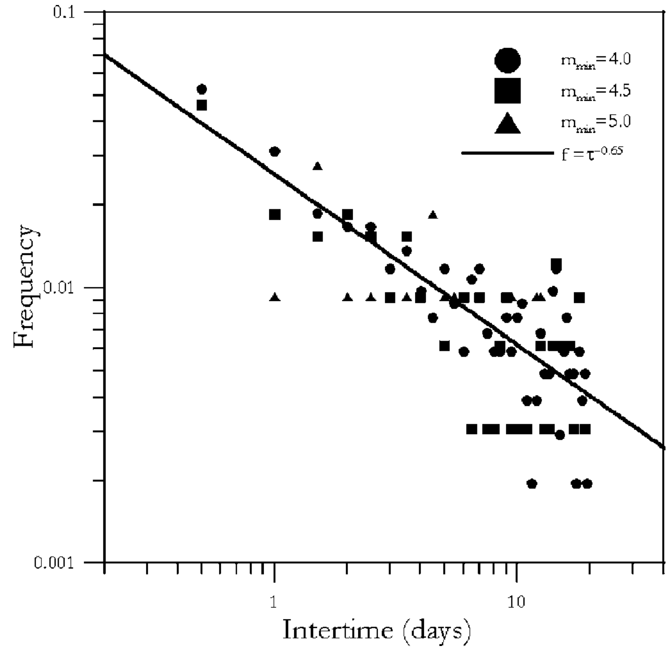

FIG. 7. Intertime distribution between large earthquakes, for different values of magnitude cutoff m_{min}.

When the bin width Δt assumes high values, the characteristic on-off intermittent exponent value ($\simeq 1.5$) is observed only for some values of the threshold. This feature seems to be in contrast with the characteristics of on-off intermittency behavior [16]. For lower threshold values we obtain higher exponents indicating that shorter laminar phases are more frequent. In this case we are looking at the clustering phenomenon zooming at a scale where the temporal correlations are very short and this is typical of chaotic behavior. Namely we are considering very small clusters as independent ones, whereas it is well known that these should be considered as an effect of a branching mechanism in the cluster generation: each earthquake can be viewed as the "mother" of other earthquakes at many generation level. A more detailed discussion of time dependent clustering degree can be found in the literature on the ETAS model [10,29–34] and the multifractal behavior of earthquake temporal occurrence [2,7,35–37]. Nevertheless, the general behavior of earthquake occurrence can be viewed as an on-off intermittent phenomenon.

Notice that the power law distribution of laminar phases can be interpreted as due to a very complex mechanism as the one provided by ETAS model and not as a simple distribution of waiting times between clusters of events. Indeed, if we evaluate the distribution of the intertime between large earthquakes, here considered as main shocks, we obtain a power law with an exponent significantly lower than 1.5 (Fig. 7). This reveals that the bursting activity is not directly linked to the magnitude of the mother earthquake confirming, again, that all earthquakes should be considered on the same foot-level.

III. CONCLUSIONS

Earthquake occurrence exhibits an evident similarity with on-off intermittency. This behavior depends on the value of the threshold and the length of the bin width Δt. For low values of the threshold, we observe a transition to chaotic behavior, whereas higher values of the bin width Δt and higher values of the threshold make the statistics too poor.

These results imply that earthquake occurrence can be considered as an example of on-off intermittency (or alternately of chaotic behavior). The typical bursting behavior of intermittent phenomena does not allow any prediction about their occurrence, confirming the difficulty in earthquake forecast, suggested by Geller *et al.* [38]. Nevertheless, the

TABLE IV. Exponent of the laminar phases distributions for $\Delta t = 600$ min.

Thresholds	Exponent	Standard deviation
5	1.9	0.4
10	1.5	0.4
15	1.4	0.5
20	1.5	0.5

TABLE V. Exponent of the laminar phases distributions for $\Delta t = 1$ day.

Thresholds	Exponent	Standard deviation
5	2.7	0.6
10	1.6	0.3
15	1.5	0.5
20	1.3	0.5

assumption of intermittency allows the great advantage to quantify the predictability degree when a sufficiently long record is available (see, e.g, Ref. [39]). Unfortunately, this is not the case because a detailed analysis of the fluctuations of the "predictability" of earthquakes is not possible with our data and further investigation of longer catalogues would be required.

As well known, we can easily obtain on-off intermittency randomly driving a logistic map. Thus, we can adopt a logistic mechanism as a model for earthquake occurrence. This provides very useful information about the dynamical behavior of seismicity. Indeed, the logistic mechanism implies that the earthquake rate of occurrence depends on its present value giving rise to the well-known logistic increasing of the population [40]. The dependence of the present value on the previous one creates a temporal correlation structure, which can be explained, for example, by means of the dependent earthquake nucleation mechanism [41].

This is the case of the SOC model with memory introduced by Lippiello *et al.* [22], where the state dependent nucleation mechanism is mimicked by the introduction of a memory effect. It is noteworthy that the mean-field approximation of this model [21] leads to a logisticlike equation, which gives rise to on-off intermittency only when the control parameter assumes a particular value sequence derived with the SOC original model. Our experimental observation is in very good agreement with the prediction of on-off intermittent behavior in earthquake occurrence, confirming again the existence of long range correlations well modeled by the memory effect.

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